The Impact of CO$_2$, H$_2$O and Other “Greenhouse Gases” on Equilibrium Earth Temperatures

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Abstract: It has long been accepted that the “greenhouse effect”, where the atmosphere readily transmits short wavelength incoming solar radiation but selectively absorbs long wavelength outgoing radiation emitted by the earth, is responsible for warming the earth from the 255K effective earth temperature, without atmospheric warming, to the current average temperature of 288K. It is also widely accepted that the two main atmospheric greenhouse gases are H$_2$O and CO$_2$. What is surprising is the wide variation in the estimated warming potential of CO$_2$, the gas held responsible for the modern concept of climate change. Estimates published by the IPCC for climate sensitivity to a doubling of CO$_2$ concentration vary from 1.5 to 4.5°C based upon a plethora of scientific papers attempting to analyse the complexities of atmospheric thermodynamics to determine their results. The aim of this paper is to simplify the method of achieving a figure for climate sensitivity not only for CO$_2$, but also CH$_4$ and N$_2$O, which are also considered to be strong greenhouse gases, by determining just how atmospheric absorption has resulted in the current 33K warming and then extrapolating that result to calculate the expected warming due to future increases of greenhouse gas concentrations. The HITRAN database of gaseous absorption spectra enables the absorption of earth radiation at its current temperature of 288K to be accurately determined for each individual atmospheric constituent and also for the combined absorption of the atmosphere as a whole. From this data it is concluded that H$_2$O is responsible for 29.4K of the 33K warming, with CO$_2$ contributing 3.3K and CH$_4$ and N$_2$O combined just 0.3K. Climate sensitivity to future increases in CO$_2$ concentration is calculated to be 0.50K, including the positive feedback effects of H$_2$O, while climate sensitivities to CH$_4$ and N$_2$O are almost undetectable at 0.06K and 0.08K respectively. This result strongly suggests that increasing levels of CO$_2$ will not lead to significant changes in earth temperature and that increases in CH$_4$ and N$_2$O will have very little discernable impact.

Keywords: Carbon Dioxide, Climate Sensitivity, Greenhouse Effect, Climate Change

1. Introduction - Equilibrium Climate Sensitivity

Equilibrium Climate Sensitivity (ECS) is generally understood to be the increase in earth surface temperature resulting from a doubling of atmospheric CO$_2$ levels after thermal equilibrium has been achieved (Zhong and Haigh 2013) [1]. Values for ECS deduced from a variety of climate models show a high degree of uncertainty with a typical range of 1.5 to 4.5°C (AR5-WG1-SPM, p16) [2]. More recent work, however, suggests ECS values of less than 1degC (Harde-2017) [3] (Wijngaarden and Happer 2020) [4].

The difficulty is that there are countless atmospheric variables which can be judged to have an impact upon this value. This situation is neatly summed up in the note by the UK MET Office, “What is Climate Sensitivity?” (metoffice.gov.uk).

“As there is no ‘perfect’ way of estimating climate sensitivity, it remains a hotly debated area of science and there...
remains a wide range of estimates of what the ECS could be.” This is an unfortunate situation since world governments are implementing ambitious and expensive plans to limit the increase of global temperatures by reduction of CO₂ emissions to atmosphere, and to achieve a net zero carbon economy, in the belief that CO₂ emissions are the main driver of global temperature increases. In order to justify such a fundamental change to the global economy and indeed the associated risk to an “established way of life” it is of paramount importance that an accurate value for ECS is determined. To do this, it will be necessary to cut a swathe through the complexities of atmospheric thermodynamics and find a simple method of accurately estimating ECS.

1.1. The “Greenhouse Effect”

The average temperature of the earth is determined solely by the energy balance at the top of the atmosphere (TOA). At equilibrium, energy input from the sun is balanced by the energy radiating from the earth, which is a function of its temperature. The term “greenhouse effect” is used to describe the process whereby short wavelength solar radiation is transmitted through the atmosphere, whereas components of the long wavelength radiation emitted by the earth are absorbed by the atmosphere, thus limiting reradiation into space with a consequential warming of the earth (Schwartz 2018) [5, 6]. The atmospheric gases which absorb this energy, principally CO₂ and H₂O, are collectively referred to as “greenhouse gases”. In order to quantify this warming, it is first necessary to identify the key contributing variables.

1) Solar Radiation. This is the radiated energy received by the earth from the sun. It may be calculated from a knowledge of the sun surface temperature and the diameter and distance of the earth from the sun. This value is corroborated by direct measurement from orbital satellites. (Trenberth et al 2009) [7].

2) Earth’s Albedo. The earth’s reflectivity to solar radiation. This is a measurement of how much of the solar radiation is reflected into space, thus weakening the solar radiation input.

3) Earth’s Emissivity. All bodies emit thermal radiation as a function of their temperature according to the Planck equation for black body emission. The emissivity of the earth determines what fraction of that black body radiation is actually emitted by the earth.

4) Atmospheric Absorption. Absorption of the long wavelength radiation emitted from the earth by atmospheric gases, in particular H₂O and CO₂, both possessing strong infra-red radiation absorption characteristics.

5) Retention of Absorbed Energy. This is the key issue which contributes much to the uncertainty in determination of climate sensitivity. What happens to the energy absorbed by the atmosphere? There are only two possibilities; some of the energy is reradiated through to space by the warmed atmosphere, or the energy is retained by the earth and its atmosphere thus contributing to the planetary warming. This energy retention is a function of complex atmospheric thermodynamics and its estimation is a major cause of the wide variations in climate sensitivity estimations.

1.2. Retained Energy

Variables 1 to 4 in the list above are all easily identifiable and may be determined with reasonable accuracy. Variable 5, Retention of Absorbed Energy, however, is in a different league. Atmospheric thermodynamics are immensely complicated and to use them to determine an accurate estimate of the fraction of absorbed energy that is retained is near impossible in the non-linear chaotic system that is the earth’s atmosphere and is the primary contributor to the wide range of estimates of climate sensitivity (Stevens B. et. al., 2016) [8]. There is, however, an alternative method of determining this retained energy factor using the calculations of earth temperature.

1.3. Effective Earth Temperature \( T_e \)

If \( I_0 \) is the energy flux received by earth from the sun, the total incident energy will be \( I_0 \pi r^2 \), where “\( r \)” is the radius of the earth. A fraction “\( f \)” of this energy will be reflected by the earth into space (earth’s albedo). The energy received by the earth is then \( (1 – f)I_0 \pi r^2 \). The energy \( R \) radiated by the earth as a result of its temperature \( T \) will be \( e \cdot E \cdot 4\pi r^2 \) (energy radiated over the complete earth surface of \( 4\pi r^2 \)), where “\( e \)” is the earth emissivity. The total energy now retained by the atmosphere/earth system and some will be reradiated by the atmosphere through to space. Let “\( n \)” represent the “energy retention factor”, the fraction of the absorbed energy \( a \cdot R \) that is retained by the atmosphere/earth system, so that the fraction transmitted to space is \( (1 – n) \cdot a \cdot R \). The total energy now transmitted to space is
Total energy transmitted is \( (1-a) \cdot R + (1-n) \cdot a \cdot R \)

\[ = (1-n \cdot a) \cdot R \]

And for equilibrium at the TOA

\[ (1-f) \cdot L_0 \cdot \pi r^2 = (1-n \cdot a) \cdot c \cdot \sigma T^4 \cdot 4\pi r^2 \]

So that equilibrium temperature \( T = \sqrt[4]{\frac{(1-f) \cdot L_0}{4 \cdot \sigma \cdot (1-n \cdot a)}} \)

Therefore

\[ T = \frac{255}{\sqrt[4]{\frac{1}{1-n \cdot a}}} \]  

(2)

since from (1), the term \( T = \sqrt[4]{\frac{(1-f) \cdot L_0}{4 \cdot \sigma \cdot (1-n \cdot a)}} \) is already established as the effective earth temperature of 255K.

This equation applies for all atmospheric conditions, including any effect due to atmospheric absorption of incoming solar radiation and the average impact of cloud cover. The temperature is influenced by just two variables, atmospheric absorptivity “\( a \)” and the energy retention factor “\( n \)”. Atmospheric absorptivity “\( a \)” may be calculated from a knowledge of gaseous composition, atmospheric “thickness” and the infra-red absorption spectra of those gases. Designating the total absorptivity of the current atmospheric condition as “\( a_0 \)” and re-arranging (2) gives the result

\[ n \cdot a_0 = (1 - (255/T)^4) \]

Without any knowledge of, or assumptions about, atmospheric thermodynamics, the value of “\( n \)” can be ascertained from the current Equilibrium Earth Temperature \( T \), usually considered to be 288K so that

\[ n \cdot a_0 = (1 - (255/288)^4) \]

\[ n \cdot a_0 = 0.385 \]

This value represents the total fraction of the radiated energy which is absorbed and retained by the earth/atmosphere system, leaving 61.5% of the radiated energy transmitted through to space (Wilson and Gea-Banacloche 2012) [9]. The energy retention factor “\( n \)” is then determined from a knowledge of the total absorptivity \( a_0 \) of the current atmosphere.

\[ n = 0.385/a_0 \]  

(3)

This result does not suggest that “\( n \)” is a function of atmospheric absorption “\( a \)”. It is simply a method of using a known boundary condition, that is the current earth temperature and atmospheric absorption “\( a_0 \)” due to the current composition of greenhouse gases, to determine the unknown variable “\( n \)”.

By determining “\( n \)” it is then possible to evaluate with some accuracy, from a knowledge of the infra-red absorption characteristics of the atmosphere, how variations in atmospheric composition, particularly CO₂, will impact on atmospheric absorptivity and consequently, the earth temperature. This results in a precise assessment of climate sensitivity not only to CO₂, but other radiation absorbing gases such as CH₄ and N₂O as well as an estimation of the feedback effects of H₂O. It is first necessary however to calculate the absorptivity “\( a_0 \)” for the current atmospheric concentrations of H₂O, CO₂, CH₄ and N₂O.

![Figure 1](image.png)

**Figure 1. Spectral radiance \( \nu \) Wavelength \( \mu m \) for a black body radiator at 288K Kelvin.**

### 2. Atmospheric Absorptivity

#### 2.1. Spectral Radiance

Incoming radiation from the sun is predominantly short wavelength visible and near infra-red radiation. This passes through the atmosphere without significant absorption. The radiation from the earth however is purely a function of its temperature and is readily calculated by the Planck equation

\[ \text{Spectral Radiance } L_\lambda = c_1 \cdot \lambda^{-5} \cdot (\exp(c_2/\lambda T) - 1) \text{ W/m}^2 \text{ m} \]  

(4)

Where \( \lambda \) is the wavelength of emitted radiation, \( c_1 \) and \( c_2 \) are the radiation constants. Figure 1 shows the spectral radiance from a body at a temperature of 288K, the average temperature of the earth’s surface. The radiation extends over a wide range of wavelengths from 3 to 100 \( \mu \text{m} \) (10⁻⁶ m).

#### 2.2. HITRAN Database

In order to assess the impact of atmospheric greenhouse gases, it is necessary to be able to calculate exactly what fraction of the radiated energy is absorbed by differing concentrations of those gases, particularly CO₂, H₂O, CH₄ and N₂O throughout this radiation spectrum. For this a web database known as HITRAN (I. E. Gordon et al 2017) [10] has been invaluable.

The HITRAN database tabulates the transmissivity of a wide variety of gases over a range of concentrations, pressure and temperature throughout this spectral range of the radiated energy. The gases of interest are principally CO₂, H₂O, CH₄ and N₂O as minor atmospheric constituents. A “greenhouse gas” is defined simply as one which can absorb infra-red energy within this spectral range of 3 to 100\( \mu \text{m} \).

The atmosphere, however, is not a uniform medium. As altitude increases, pressure, temperature and gas density all decrease, with the result that the transmissivity of infra-red
radiation through the atmosphere varies with altitude. Additionally, H$_2$O concentrations are a function of the saturated vapour pressure (SVP) of water which determines how much gaseous H$_2$O can be held by the atmosphere. SVP is also a function of temperature and therefore of altitude.

2.3. Atmospheric Altitude Variation

The variation with altitude of pressure, temperature and SVP are calculated using the following formulae.

2.3.1. Temperature

Temperature reduces at the adiabatic lapse rate of 6.5K/km to a temperature of 216K at 11000m and then remains relatively constant to 35000m.

2.3.2. Pressure

Atmospheric pressure variation with altitude can be represented by the approximation

$$P(alt)=P_0(1-\frac{L}{T_0})(\frac{gM}{R})$$

Where $P(alt)$ is Pressure at altitude (pascals) $P_0$ is Pressure at sea surface (pascals) $L$ is Adiabatic Lapse Rate (-0.0065K/m) $h$ is Altitude (m) $g$ is Acceleration due to gravity (9.807m/s$^2$) $M$ is Molar mass of dry air (0.029kg/mol) $R$ is Gas Constant (8.314J/mol.K)

See Figure 2 (US Standard Atmosphere 1976) [11].

2.3.3. H$_2$O Concentration

The concentration of water vapour is governed by the Saturation Vapour Pressure (SVP) which is a unique function of temperature. SVP is the maximum amount of water vapour that can exist without precipitating out into liquid water, i.e. cloud and rain. Actual average values of water vapour concentration are always lower than the SVP and are usually represented by the term Relative Humidity (RH) which is % of SVP. A value of 80%RH is generally assumed for average atmospheric water content, although local levels can be found across the globe varying anywhere from almost zero to 100%.

2.3.4. Saturation Vapour Pressure (SVP)

The following empirical “Buck” equation has been used to calculate SVP(kPa) (A. L. Buck 1981) [12].

$$SVP=0.61121\exp((18.678 – \frac{T}{234.5})(\frac{T}{257.14 + T}))$$

where T is temperature in degree Celsius

Figure 3 shows the variation of water vapour concentration with an RH of 80% at atmospheric altitudes up to 35000m. As can be seen water vapour concentration drops to almost zero above 10,000m.

2.3.5. CO$_2$, CH$_4$ and N$_2$O Concentration

Unlike water vapour, the mean CO$_2$ concentration will remain constant at all atmospheric levels, although its density will reduce as altitude increases and pressure and temperature decrease. CO$_2$ concentration however will vary considerably with location and with seasons, as biospheric photosynthesis removes substantial seasonal amounts of CO$_2$ from the atmosphere. A mean level of 400ppm has been assumed for the following calculations of atmospheric absorptivity. Similarly, CH$_4$ and N$_2$O concentrations will be considered to remain constant at current average levels of 1.8ppm and 0.32ppm respectively.

2.4. Calculation of Absorptivity

As can be seen from the previous graphs, pressure, temperature and water vapour concentration vary with altitude, particularly in the lower levels below 10,000m. In order to take account of these variations, the HITRAN database has been used to calculate the transmission of radiated energy in the wavelength range 3 to 100µm through each 100m section of the atmosphere from 0 to 35000m altitude, taking into account the variations of temperature, pressure and SVP.

Let $t_{\lambda,CO_2,h}$ represent the transmissivity of the gas CO$_2$ at a specific wavelength $\lambda$ at the altitude “h”. Similar representations will be made for the other gasses’ H$_2$O, CH$_4$, N$_2$O. The values of $t_{\lambda,CO_2,h}$ for the temperature and pressure conditions at the specific altitude h and CO$_2$ concentration of 400ppm are determined from the HITRAN database. The aggregate transmissivity $t_{\lambda,h}$ to altitude level h at the wavelength $\lambda$ is then found by:
The combined transmissivity at wavelength $\lambda$ for multiple gases is found from

$$t_{\lambda, CO_2, H_2O, CH_4, N_2O, h_n} = \Pi_m (t_{\lambda, CO_2, h_m} \cdot t_{\lambda, H_2O, h_m} \cdot t_{\lambda, CH_4, h_m} \cdot t_{\lambda, N_2O, h_m})$$

for $m=1$ to $n$

HITRAN data is presented in terms of wavenumber, which is the reciprocal of wavelength, in units of $\text{cm}^{-1}$, a preferred unit in the field of spectroscopy. To obtain the total transmissivity over the wavenumber band $10$ to $3333 \text{cm}^{-1}$ (equivalent to $3$ to $100 \mu\text{m}$ wavelength), it is necessary to integrate the $t_{\lambda}$'s over the entire waveband of $10$ to $3333 \text{cm}^{-1}$.

It is also important to decide the resolution in terms of wavenumber for the determination of the values of “$t_{\lambda}$”. The higher the resolution the more accurate will be the final answers, but at the expense of increased computational resource. There is a resolution limit however in that pressure broadening of the individual absorption lines tends to smooth the spectral lines as resolution increases. A compromise spectral resolution of $0.1 \text{cm}^{-1}$ has been used for these calculations (Harde 2013) [13, 14]. To determine the actual transmission of radiated energy at the wavelength $\lambda$ through the atmosphere it is a matter of multiplying the transmissivities $t_{\lambda}$ by the spectral radiance $L_{\lambda}$ (4).

$E_{\lambda} = \Pi_m (t_{\lambda, CO_2, h_m} \cdot t_{\lambda, H_2O, h_m} \cdot t_{\lambda, CH_4, h_m} \cdot t_{\lambda, N_2O, h_m} \cdot L_{\lambda})$ W/m$^3$ for $m=1$ to 350 (35000m)

The total energy transmitted $E_E$ through the atmosphere is determined by integrating the radiation $E_{\lambda}$ over the wavelength range $3$ to $100 \mu\text{m}$

$$E_E = \sum_{\lambda} E_{\lambda} \Delta\lambda \text{ W/m}^2$$

where $\Delta\lambda$ is the wavelength increment at wavelength $\lambda$, equivalent to the wavenumber increment of $0.1 \text{cm}^{-1}$, which is the resolution chosen for the HITRAN data. The atmospheric transmissivity is then calculated from $E_E/\sigma T^4$.

$\sigma$ is the Stefan-Boltzmann constant.

2.5. Atmospheric Absorption Spectra for $CO_2$ and $H_2O$

Figures 4, 5 and 6 show the transmission of the spectral radiation $E_{\lambda}$ through current atmospheric concentrations of $CO_2$ and $H_2O$ and through the combination of the two gases. Absorptivities of both $CO_2$ and $H_2O$, as well as $CH_4$ and $N_2O$, have been determined over the range $3$ to $100 \mu\text{m}$ to a resolution of $0.1 \text{cm}^{-1}$. It is clear that significant amounts of radiated energy are absorbed by both $CO_2$ and $H_2O$. It is also clear that there is considerable overlap of the absorption bands of $CO_2$ and $H_2O$ with the $H_2O$ absorption being the dominant factor.

It is also possible to calculate the absorption of energy as it
passes from sea level to 35000m as shown in Figure 7. As can be seen, most of the radiation absorption occurs in the lower 5000m of the atmosphere. Beyond that point the radiation in the CO$_2$ and H$_2$O absorption bands has been absorbed and reduced to almost zero. Little further absorption can occur. The calculations and this graph show the total atmospheric absorptivity due to H$_2$O and CO$_2$ to be 72.6%, with the dominant absorber, H$_2$O, contributing 67.0% while the impact of 400ppm CO$_2$ provides the additional 5.6%. Note that without the presence of H$_2$O, the absorption due to CO$_2$ would be 18%. The overlapping spectral absorption bands of the dominant H$_2$O molecules reduce the CO$_2$ impact to 5.6%.

### 2.6. Impact of Methane (CH$_4$) and Nitrous Oxide (N$_2$O)

CH$_4$ and N$_2$O are recognized as strong absorbers of long wavelength infra-red energy and are considered to be significant contributors towards global warming. It is claimed that CH$_4$ has a global warning potential 86 times stronger per unit mass than CO$_2$ (Etminan, Myhre, Highwood and Shine, 2016) [15].

#### 2.6.1. Current Atmospheric Concentrations 1.8ppm CH$_4$ and 0.32ppm N$_2$O

The spectral transmissions of 1.8ppm CH$_4$ and 0.32ppm N$_2$O, are also calculated using HITRAN data base and are shown in Figures 8 and 9. All the infra-red absorption for these two gases occurs at wavelengths below 20µm. When total atmospheric infra-red transmission, including.

\[ \text{Figure 8. Spectral Transmission through the atmosphere of 1.8ppm CH}_4 \text{ v Wavelength } \mu\text{m.} \]

\[ \text{Figure 9. Spectral Transmission through the atmosphere of 0.32ppm N}_2\text{O v Wavelength } \mu\text{m.} \]

CH$_4$ and N$_2$O is shown in figure 10, superimposed over the transmission through H$_2$O and CO$_2$ alone, it is immediately clear that the impact of both CH$_4$ and N$_2$O is almost completely swamped by the combined overlapping absorption bands of H$_2$O and CO$_2$. Nevertheless, there is a small measurable effect in that the total atmospheric absorptivity is increased from 72.6% to 73.0%, an increase of 0.4%, by the contribution of these two gases at current atmospheric concentrations.

\[ \text{Figure 10. Total Spectral Transmission CH}_4 + N_2O + CO_2 + H_2O \text{ through the atmosphere v Wavelength } \mu\text{m.} \]

\[ \text{Figure 11. Atmospheric absorptivity variation with CH}_4 \text{ and N}_2\text{O concentration.} \]

#### 2.6.2. Increasing Concentrations of CH$_4$ and N$_2$O

CH$_4$ and N$_2$O are indeed very powerful absorbers of infra-red radiation. Increasing the concentrations of each gas to 30ppm (a 16fold increase in the case of CH$_4$ and an almost 100fold increase in N$_2$O) would result in a combined absorption of 15%, close to the value of 18% for 400ppm of CO$_2$. The combined absorptive impact in the presence of H$_2$O and CO$_2$ however reduces this absorption to less than 3% as can be seen in Figure 11 due to the overlap of the absorption bands of CO$_2$ and H$_2$O. It would thus take a huge increase in atmospheric concentrations of these gases to have any significant impact on total atmospheric infra-red absorption.

### 2.7. Retention of Absorbed Energy

The calculated value of 0.730 for the absorption of the earth’s radiated energy for the current average atmospheric
condition (400ppm CO$_2$, 80% Relative Humidity, 1.8ppm CH$_4$, 0.32ppm N$_2$O and temperature 288K), allows us to determine, without any assumption or even knowledge of the complex atmospheric thermodynamics, the fraction “n” of absorbed energy which is retained by the earth.

\[ \text{From (3)} \quad n = \frac{0.385}{a_0} \]

\[ \text{While} \quad a_0 = 0.730 \]

\[ \text{Thus} \quad n = 0.5274 \]

This suggests that only 52.74% of the radiated energy absorbed by the atmosphere is actually retained by the earth/atmosphere system. So, the net retention of the radiated energy is 52.74% of 73.0% = 38.5%, leaving 61.5% of radiated energy being transmitted to space to balance the solar energy input and produce an average earth temperature of 288 Kelvin. The total absorption and transmission of radiated energy is summarised in the pie chart of Figure 12.

By calculating the absorptivity “a” for a range of atmospheric gas concentrations using the HITRAN database, the earth temperature \( T \) can now be determined from (2) by substituting \( n = 0.5274 \), so that

\[ T = \sqrt{\frac{255}{n} - 0.5274} \text{ Kelvin} \quad (6) \]

allowing an accurate assessment of climate sensitivity to variations in atmospheric absorptivity and hence to concentration of “greenhouse gases”.

| Table 1. Variation of atmospheric absorptivity with CO$_2$ concentration. |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \text{CO}_2 \) ppm      | 0               | 10              | 50              | 100             | 200             | 400             | 800             | 1600            |
| \( \text{CO}_2 \) Absorption | %              | %               | %               | %               | %               | %               | %               | %               |
| \( \text{Total Absorption} \) | %              | %               | %               | %               | %               | %               | %               | %               |
| 0.0                        | 9.7            | 13.5            | 15.2            | 17.0            | 18.7            | 20.4            | 22.3            | 24.6            |
| 67.7                       | 70.2           | 71.2            | 71.8            | 72.4            | 73.0            | 73.0            | 74.6            | 75.0            |

3. Atmospheric Impact on Equilibrium Earth Temperatures

3.1. Variation of Total Absorptivity with Atmospheric CO$_2$ Concentration

Repeating the calculation procedure using HITRAN data for CO$_2$ concentrations from 50ppm through to 1600ppm results in the data in Table 1 and Figure 13 for total absorption which includes the absorption due to H$_2$O, 1.8ppm CH$_4$ and 0.32ppm N$_2$O.

3.2. Variation of Equilibrium Earth Temperature with CO$_2$ Concentration

Using (5) \( T = \sqrt{\frac{255}{1 - 0.5274/a_0}} \) Kelvin

The corresponding temperature changes, as absorptivity varies with CO$_2$ concentration, are as shown in the following Table 2 and Figure 14. Increasing CO$_2$ concentrations from the current 400ppm to 1600ppm will increase global temperatures from 288K to 289K, an increase of just 1 Kelvin.

| Table 2. Variation of earth equilibrium temperature with CO$_2$ concentration, assuming all other atmospheric constituents remain at their current values. |
|-----------------|---------|--------|--------|--------|--------|--------|--------|--------|
| \( \text{CO}_2 \) ppm | 10     | 50     | 100    | 200    | 400    | 800    | 1600   |
| Temperature Kelvin | 284.8  | 286.2  | 286.9  | 287.6  | 288.0  | 288.4  | 289.0  |
3.3. Variation of Equilibrium Earth Temperature with CH$_4$ and N$_2$O Concentration

3.3.1. CH$_4$ Methane

The absorptivity of methane in the atmosphere at concentrations up to 28.8ppm, 16 times current atmospheric levels, is calculated using HITRAN data and is shown in Table 3. Also, in Table 3 are the total atmospheric absorption levels, including the impact of current atmospheric levels of CO$_2$, H$_2$O and N$_2$O and the resulting equilibrium temperature calculated using (5).

<table>
<thead>
<tr>
<th>ppm CH$_4$</th>
<th>1.8</th>
<th>3.6</th>
<th>7.2</th>
<th>14.4</th>
<th>28.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH$_4$ absorption %</td>
<td>1.6</td>
<td>2.1</td>
<td>2.8</td>
<td>3.7</td>
<td>4.7</td>
</tr>
<tr>
<td>Total absorption %</td>
<td>73.03</td>
<td>73.1</td>
<td>73.2</td>
<td>73.36</td>
<td>73.54</td>
</tr>
<tr>
<td>Temperature Kelvin</td>
<td>287.97</td>
<td>288.02</td>
<td>288.09</td>
<td>288.17</td>
<td>288.28</td>
</tr>
</tbody>
</table>

This is, at first sight, a surprisingly low temperature variation, until the realisation sets in that the main H$_2$O absorption bands are already well saturated, in that almost all the infra-red energy in those bands has already been absorbed. Changes in H$_2$O concentration have little further impact.

3.3.2. N$_2$O Nitrous Oxide

Similar HITRAN calculations for N$_2$O concentrations up to 5.1ppm, again 16 times current atmospheric levels of 0.32ppm, are provided in Table 4 showing the impact on absorptivity and temperature.

<table>
<thead>
<tr>
<th>ppm N$_2$O</th>
<th>0.32</th>
<th>0.64</th>
<th>1.28</th>
<th>2.56</th>
<th>5.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>N$_2$O absorption %</td>
<td>1.7</td>
<td>2.6</td>
<td>3.8</td>
<td>5.4</td>
<td>7.3</td>
</tr>
<tr>
<td>Total absorption %</td>
<td>73.03</td>
<td>73.14</td>
<td>73.30</td>
<td>73.54</td>
<td>73.87</td>
</tr>
<tr>
<td>Temperature Kelvin</td>
<td>287.97</td>
<td>288.04</td>
<td>288.14</td>
<td>288.28</td>
<td>288.49</td>
</tr>
</tbody>
</table>

While both CH$_4$ and N$_2$O are both recognised as strong absorbers of infra-red radiation their impact is muted by the dominant H$_2$O radiation absorption to the point that an unlikely 16fold increase in concentrations would result in an increase in the earth’s temperature of 0.3K due to CH$_4$ and 0.5K due to N$_2$O.

3.4. The Impact of H$_2$O

The maximum H$_2$O concentration that can be held in the atmosphere is a direct consequence of temperature and is governed by the saturated vapour pressure. Actual H$_2$O concentrations are mostly lower than this maximum value, represented by the fraction “Relative Humidity” RH. An RH of 0.8 has been assumed throughout for these calculations. It would be useful to determine what impact variations in RH will have upon atmospheric absorption and earth temperatures. The results of calculations of RH variations from 0.6 through to 1.0 are provided in Table 5 and Figures 15 and 16. A variation in Relative Humidity of +/- 20% results in a variation of average earth temperature of +/- 0.6K.

<table>
<thead>
<tr>
<th>Relative Humidity %</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$O absorption %</td>
<td>65.1</td>
<td>66.1</td>
<td>67.0</td>
<td>67.8</td>
<td>68.6</td>
</tr>
<tr>
<td>Total absorption %</td>
<td>72.0</td>
<td>72.5</td>
<td>73.0</td>
<td>73.5</td>
<td>73.9</td>
</tr>
<tr>
<td>Temperature Kelvin</td>
<td>287.3</td>
<td>287.7</td>
<td>288.0</td>
<td>288.2</td>
<td>288.5</td>
</tr>
</tbody>
</table>

3.5. Water Vapour Feedback

It is often claimed that increasing temperatures due to increasing CO$_2$ levels will result in a positive feedback since the SVP of H$_2$O will also increase thus enabling the atmosphere to retain higher levels of moisture, giving rise to significantly higher climate sensitivity values (Held and Soden 2000) [16].

Assume a temperature increase $\Delta T_0$ for whatever reason. That increase will result in an increase in H$_2$O saturated vapour pressure and an increase in H$_2$O concentration $\Delta$H$_2$O, proportional to the temperature increase. $\Delta$H$_2$O = $j \cdot \Delta T_0$ where “j” is the concentration proportionality constant.

In turn, the increased concentration will increase infra-red absorption resulting in a further increase in temperature $\Delta T_1$ proportional to $\Delta$H$_2$O.

$\Delta T_1 = k \cdot \Delta$H$_2$O “k” is the temperature proportionality constant.

Thus $\Delta T_1 = j \cdot k \cdot \Delta T_0$

Of course, this temperature increase $\Delta T_1$ will itself produce a further temperature increase $\Delta T_2$ where

$\Delta T_2 = j \cdot k \cdot \Delta T_1 = (j \cdot k)^2 \Delta T_0$

And so on, resulting in a total temperature increase

$\Delta T = \Delta T_0 + \Delta T_1 + \Delta T_2 + \ldots + \Delta T_n + \ldots$
\[ \Delta T = \Delta T_0 \sum_{n=0}^{\infty} (j \cdot k)^n \]\n
\( \sum_{n=0}^{\infty} (j \cdot k)^n \) is an infinite series that, if \((j \cdot k) \geq 1\), will diverge and the atmospheric system will be unstable, and the earth uninhabitable. If \((j \cdot k) < 1\) the series will converge, and the impact of \(H_2O\) will be to amplify the original temperature increase \(\Delta T_0\). The question now is by how much?

To answer this, numerical values must be allocated to the coefficients, “\(j\)” and “\(k\)”. Differentiating the equation for SVP (5) with respect to \(T\), provides values for the SVP sensitivity to temperature \(d(SVP)/dT\) (Figure 17).

From this data for SVP variation with temperature a value for “\(j\)” can be allocated. Since temperature, and thus SVP reduce with altitude, the sensitivity of SVP to changes in temperature \(d(SVP)/dT\) also varies with altitude. However, from Figure 17 it is easy to see that the lower altitude levels of the atmosphere have by far the strongest effects on absorptivity. A realistic, and indeed worst case, value for the constant “\(j\)” would be the value at sea level, which is \(j = 0.088\%/K\).

For “\(k\)” it is necessary to look first at Figure 16, which shows temperature change for variations in RH value. Specifically, it is clear that an RH variation from 0.6 to 1.0 results in an almost linear temperature change of 1.2K. From Figure 4 the \(H_2O\) concentration at sea level assuming 80% RH is 1.36%. Thus a change of RH from 0.6 to 1.0 results in a change in concentration of 0.68%.

Thus \(k = 1.2K/0.68\% = 1.765\ K/%H_2O\)
And so, \(j \cdot k = 0.088 \times 1.765 = 0.155\)

This results in the infinite series \(\sum_{n=0}^{\infty} (j \cdot k)^n\) converging to a value of 1.183, which becomes an amplification factor due to \(H_2O\) for any change in temperature in Kelvin, for any reason.

### 3.6. Temperature Feedback

There is a further feedback which is often overlooked. As temperature increases the Planck radiation emitted by the earth increases proportionally to \(T^4\). The peak intensity also moves towards shorter wavelengths. This changes slightly the overall absorption by the greenhouse gases, since their absorptivity is also wavelength dependent. Figure 18 shows the total atmospheric absorption variation with earth temperature. It is an almost linear relationship with a negative gradient. As temperature increases the absorptivity decreases.

This leads to a negative feedback effect, not dissimilar to the \(H_2O\) feedback effect, but in the opposite direction. The decrease in absorptivity produces a corresponding decrease in temperature, and so on. Figure 19 gives the relationship for the temperature decrease produced by a decrease in absorptivity.

\[ \Delta T = \Delta T_0 \sum_{n=0}^{\infty} (p \cdot q)^n \]  

Where “\(p\)” is the slope of the absorption v temperature in Figure 18 in %/Kelvin, and “\(q\)” is the slope of the temperature increase resulting from an increase in absorptivity in Figure 19 in Kelvin/%.

\[ p = -0.0855 \text{ %/Kelvin} \]
\[ q = 0.6173 \text{ Kelvin/%} \]

resulting in a value for \(p \cdot q\) of -0.0528 and a value, for the infinite series \(\sum_{n=0}^{\infty} (p \cdot q)^n\) of 0.9498, a feedback factor which actually opposes the \(H_2O\) feedback. A total feedback multiplier is found by multiplying the two contributing feedback factors

\[ 0.9498 \times 1.183 = 1.124 \]

thus reducing the feedback effect of \(H_2O\) by one third from an increase of 18.3% to one of 12.4%.

![Figure 17. Rate of change of Saturated Vapour Pressure with Temperature v Altitude.](image)

![Figure 18. Variation of Total Absorptivity with Earth Radiation Temperature. Slope=0.0855%absorption/Kelvin.](image)

![Figure 19. Variation of Temperature with Atmospheric Absorptivity Slope=0.6173 K%/absorption.](image)
4. Equilibrium Climate Sensitivity

It is now possible to infer meaningful and accurate values for climate sensitivities to all "greenhouse gases" defined as the change in average earth temperature upon reaching equilibrium from the effects of a doubling of the gas concentration.

4.1. Climate Sensitivity to CO$_2$

From section 3.2 Table 2 shows the variation of equilibrium temperature for successive doublings of CO$_2$ concentration from 50ppm through to 1600ppm. The corresponding changes in temperature for each doubling are readily calculated in Table 6 and are shown in Figure 20. As can be seen "climate sensitivity" is not constant, but slowly increases with increasing CO$_2$ concentrations. Nevertheless, the values indicate that climate sensitivity at current CO$_2$ levels (400ppm) is of the order of 0.45 Kelvin. Applying the combined feedback H$_2$O and temperature multiplying factor of 1.124, increases the CO$_2$ climate sensitivity to 0.50 Kelvin, still significantly lower than most published values.

Table 6. CO$_2$ Climate Sensitivity.

<table>
<thead>
<tr>
<th>CO$_2$ ppm doubling</th>
<th>50-100</th>
<th>100-200</th>
<th>200-400</th>
<th>400-800</th>
<th>800-1600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate sensitivity K</td>
<td>0.34</td>
<td>0.38</td>
<td>0.41</td>
<td>0.45</td>
<td>0.54</td>
</tr>
<tr>
<td>Sensitivity x feedback K</td>
<td>0.38</td>
<td>0.42</td>
<td>0.46</td>
<td>0.50</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Figure 20. Climate Sensitivity to CO$_2$.

4.2. Climate Sensitivity to CH$_4$ and N$_2$O

Section 3.3 provides the data for changes in earth temperature for successive doublings of the concentrations of CH$_4$ and N$_2$O.

Methane CH$_4$- Table 7 and Figure 21

Table 7. CH$_4$ Climate Sensitivity.

<table>
<thead>
<tr>
<th>CH$_4$ ppm doubling</th>
<th>1.8-3.6</th>
<th>3.6-7.2</th>
<th>7.2-14.4</th>
<th>14.4-28.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate sensitivity K</td>
<td>0.05</td>
<td>0.07</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>Sensitivity x feedback K</td>
<td>0.06</td>
<td>0.08</td>
<td>0.10</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Figure 21. Climate Sensitivity to CH$_4$.

Nitrous Oxide N$_2$O:-Table 8 and Figure 22

Table 8. N$_2$O Climate Sensitivity.

<table>
<thead>
<tr>
<th>N$_2$O ppm doubling</th>
<th>0.32-0.64</th>
<th>0.64-1.28</th>
<th>1.28-2.56</th>
<th>2.56-5.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate sensitivity K</td>
<td>0.07</td>
<td>0.10</td>
<td>0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>Sensitivity x feedback K</td>
<td>0.08</td>
<td>0.11</td>
<td>0.16</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Figure 22. Climate Sensitivity to N$_2$O.

It is clear that climate sensitivity for these two gases is not a constant. It increases with concentration, for the simple reason that at these low concentrations the main radiation absorption bands are not yet saturated. At current atmospheric levels of 1.8ppm CH$_4$ and 0.32ppm N$_2$O the current climate sensitivities including feedback effects are 0.06 and 0.08 Kelvin respectively. While both gases are described as strong greenhouse gases their actual impact upon earth temperatures at current concentration levels is actually so small as to be almost unmeasurable.

5. Other Considerations

5.1. The Impact of Clouds

The obvious impact of clouds is to increase the reflectivity of the earth thus reducing the level of incoming solar radiance $I_0$ which will have a cooling influence on the earth. However, this paper is concerned with the effects of retained IR emissions from the earth.

Cloud cover will not affect the absorbance of atmospheric greenhouse gases, but it will impact upon the total energy
absorbed and possibly upon the energy retention factor “n”, in ways that would be difficult to quantify. However, by using the current average earth condition, which includes cloud, as a calibration point to determine an effective value for “n” consistent with the mean earth temperature of 288K, the current average impact of cloud has, in effect, already been taken into account. This however does not, in itself, identify what this impact is.

The structure of clouds is diverse and complex. It is close to impossible to derive a set of equations to describe the formation, structure and impact of clouds on the retention of absorbed energy and hence the radiative balance of the earth. This paper has so far relied upon the extensive HITRAN spectral database, basic physics and simple mathematics to determine values for climate sensitivity. Any attempts to estimate the further impact of cloud would be based upon speculation only and would not be appropriate for this paper.

5.2. Effect of Recently Increased Atmospheric CO$_2$

It is of some interest to calculate the increase in temperature that has occurred due to the increase in atmospheric CO$_2$ levels from the 280ppm prior at the start of the industrial revolution to the current 420ppm registered at the Mona Loa Observatory. (K. W. Thoning et. al. 2019) [17]. The HITRAN calculations show that atmospheric absorptivity has increased from 0.727 to 0.730 due to the increase of 140ppm CO$_2$, resulting in a temperature increase of 0.24Kelvin. This is, therefore, the full extent of anthropogenic global warming to date.

6. Conclusions

In order to satisfy radiative equilibrium at the “top of the atmosphere” (TOA) at an average earth temperature of 288Kelvin, only 61.5% of the earth’s radiated energy should be transmitted through to space, leaving 38.5% to be absorbed and retained by the atmosphere/earth. Use of the HITRAN data base of gaseous absorption spectra shows the current atmospheric absorption to be 73.0% of total radiative emissions of which 52.74% must be retained by the earth/atmosphere to satisfy the current TOA equilibrium. This is a simple expression of the current earth temperature equilibrium.

The 38.5% retained radiation absorption comprises 35.3% attributed to H$_2$O, 3.0% to CO$_2$ and a mere 0.2% to CH$_4$ and N$_2$O combined. From this it follows that the 33Kelvin warming of the earth from 255Kelvin, widely accepted as the zero-atmosphere earth temperature, to the current average temperature of 288Kelvin, is a 29.4K increase attributed to H$_2$O, 3.3K to CO$_2$ and 0.3K to CH$_4$ and N$_2$O combined. H$_2$O is by far the dominant greenhouse gas, and its atmospheric concentration is determined solely by atmospheric temperature. Furthermore, the strength of the H$_2$O infra-red absorption bands is such that the radiation within those bands is quickly absorbed in the lower atmosphere resulting in further increases in H$_2$O concentrations having little further effect upon atmospheric absorption and hence earth temperatures. An increase in average Relative Humidity of 1% will result in a temperature increase of 0.03Kelvin.

By comparison CO$_2$ is a bit player. It however does possess strong spectral absorption bands which, like H$_2$O, absorb most of the radiated energy, within those bands, in the lower atmosphere. It also suffers the big disadvantage that most of its absorption bands are overlapped by those of H$_2$O thus reducing greatly its effectiveness. In fact, the climate sensitivity to a doubling of CO$_2$ from 400ppm to 800ppm is calculated to be 0.45 Kelvin. This increases to 0.50 Kelvin when feedback effects are taken into account. This figure is significantly lower than the IPCC claims of 1.5 to 4.5 Kelvin.

The contribution of CH$_4$ and N$_2$O is miniscule. Not only have they contributed a mere 0.3Kelvin to current earth temperatures, their climate sensitivities to a doubling of their present atmospheric concentrations are 0.06 and 0.08 Kelvin respectively. As with CO$_2$ their absorption spectra are largely overlapped by the H$_2$O spectra again substantially reducing their impact.

It is often claimed that a major contributor to global warming is the positive feedback effect of H$_2$O. As the atmosphere warms, the atmospheric concentration of H$_2$O also increases, resulting in a further increase in temperature suggesting that a tipping point might eventually be reached where runaway temperatures are experienced. The calculations in this paper show that this is simply not the case. There is indeed a positive feedback effect due to the presence of H$_2$O, but this is limited to a multiplying effect of 1.183 to any temperature increase. For example, it increases the CO$_2$ climate sensitivity from 0.45K to 0.53K.

A further feedback, however, is caused by a reduction in atmospheric absorptivity as the spectral radiance of the earth’s emitted energy increases with temperature, with peak emissions moving slightly towards lower radiation wavelengths. This causes a negative feedback with a temperature multiplier of 0.9894. This results in a total feedback multiplier of 1.124, reducing the effective CO$_2$ climate sensitivity from 0.53 to 0.50 Kelvin.

Feedback effects play a minor role in the warming of the earth. There is, and never can be, a tipping point. As the concentrations of greenhouse gases increase, the temperature sensitivity to those increases becomes smaller and smaller. The earth’s atmosphere is a near perfect example of a stable system. It is also possible to attribute the impact of the increase in CO$_2$ concentrations from the pre-industrial levels of 280ppm to the current 420ppm to an increase in earth mean temperature of just 0.24Kelvin, a figure entirely consistent with the calculated climate sensitivity of 0.50 Kelvin.

The atmosphere, mainly due to the beneficial characteristics and impact of H$_2$O absorption spectra, proves to be a highly stable moderator of global temperatures. There is no impending climate emergency and CO$_2$ is not the control parameter of global temperatures, that accolade falls to H$_2$O. CO$_2$ is simply the supporter of life on this planet as a result of the miracle of photosynthesis.
Symbols and Abbreviations

- $a$: Atmospheric absorptivity
- $a_0$: Current atmospheric absorptivity
- $c_1$: 1st radiation constant
- $c_2$: 2nd radiation constant
- $e$: Earth emissivity
- $E$: Emissive Power $W/m^2$
- $E_\lambda$: Energy transmitted through atmosphere at wavelength $\lambda$
- $E_T$: Total energy transmitted through the atmosphere
- $f$: Earth albedo
- $h$: Altitude level (in 100m sections)
- $I_0$: Solar Irradiance $W/m^2$
- $IPCC$: Intergovernmental Panel on Climate Change
- $j$: H2O concentration proportionality constant %/K
- $k$: Temperature proportionality constant K/%
- $K$: Kelvin
- $L$: Spectral Radiance $W/m^3$
- $\rho$: Absorption temperature coefficient %/K
- $ppm$: Concentration in parts per million
- $q$: Variation of temperature with absorption K/%
- $r$: Earth radius m
- $R$: Earth radiated emission $W/m^2$
- $t$: Transmissivity
- $t_E$: Total energy transmitted $W/m^2$
- $T$: Earth temperature Kelvin
- $T_e$: Effective earth temperature Kelvin
- $TOA$: Top of the atmosphere
- $\Delta H$: H2O concentration incremental increase %
- $\lambda$: Radiation wavelength $\mu m$
- $\sigma$: Stefan’s Constant $W/m^2/K^4$

References


